# State of Charge Estimation of Sealed Lead-Acid Batteries used for Electric Vehicles

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Abstract— The various estimation methods for the state of charge of the sealed lead-acid battery have been proposed. However, any method can not accurately predict the residual capacity.

A new equation of estimating the residual capacity of the sealed lead-acid battery is proposed. The experiments were performed for verification of the proposal estimation.

Keywords-State of charge,Lead-Acid Battery.

### **1** INTRODUCTION

The electric vehicles(EV) are not yet widely accepted for replacement with the internal combustion engine car. One reason is the short driving rang of the EV with the present battery technology. This problem makes drivers afraid of exhausting all of the battery energy before the arrival at the destination.

This anxiety is solved by the exact estimation of the state of charge or the residual capacity. As a result, the excess discharge and the excess charge are avoided, and a life of the battery becomes longer.

This paper proposes one estimation method of the residual capacity(or state of charge) of the sealed lead-acid battery based on the internal resistance and electromotive force.

2 THEORY OF GENERATION OF ELECTRICITY

The lead-acid battery consists of  $PbO_2, Pb$  and  $H_2SO_4$ .

The electromotive reaction is expressed as (1), (2).

$$PbO_2 + 4H^+ + SO_4^{2-} + 2e^- \rightarrow PbSO_4 + 2H_2O$$
 (1)  
 $Pb + SO_4^{2-} \rightarrow PbSO_4 + 2e^-$  (2)

These reaction cause the discharge. And the electromotive force is expressed as (3)[1].

$$E = 2.04 + 0.0591 \log a_{H_2SO_4} / a_{H_2O} \tag{3}$$

Equation (1) and (2) show that the discharge reaction reduces  $H_2SO_4$  and increases  $H_2O$ . Thus the density of the electrolyte decreases, as the residual capacity decreases. Equation (3) indicates the electromotive force (E) decreases, as the residual capacity decreases.

# 3 Assumptions for Residual Capacity Estimation Experimental Set-up for Discharge and charge

The proposed residual capacity estimation was verified by the experiments. Thus, in this section the assumptions and conditions for the experimental set-up are described.

## 3.1 Tested Battery

We used the 24 Ah sealed lead-acid battery.

### 3.2 Equipment of Experiments

3.2.1 Equipments and Assumptions for Discharge

Fig.1 shows the equipment for discharge. The battery is discharged by the chopper (switching frequency about 20 kHz), using a resistance as a load. The battery is discharged at a constant current till the internal voltage becomes 10.7 [V]. The capacity is determined by the product of the current and the duration of discharge. And the residual capacity is determined by subtraction of the discharged capacity from the whole capacity, which is assumed the total discharged energy when the internal voltage decreases to 10.7 [V].

In this paper, the battery was discharged with three pattern of a constant current. Each date in the figure of experimental result is titled as Table I. In TABLE I, the pattern of discharge: 3 is the change

TABLE I PATTERN OF DISCHARGE CURRENT

discharge current	title in experiment result
about 4A	discharge:1,discharge:2
4A and 2A	discharge:3
about 2A	discharge:4

of 4A and 2A every 5 minutes.

### 3.2.2 Equipments and Assumptions for Charge

Fig.2 shows the equipment for the charge. The battery is charged by the same chopper. The battery is charged at a constant current (about 2.4A) till the amount of the charge, which is determined by the

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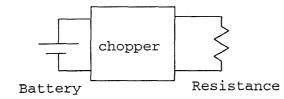


Fig. 1. Equipment for Experiment of Discharge

product of the current and the duration of discharge, becomes 26 Ah.

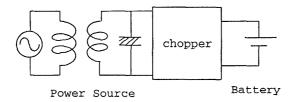


Fig. 2. Equipment for Experiment of Charge

#### **4** CONVENTIONAL ESTIMATIONS

The following estimation methods of the sealed lead-acid battery are known in [3], [4], [5] and [6].

- Estimation based on the electromotive force [3]
- Estimation based on the current integration [6]
- Estimation based on the internal resistance [4] [6]

# 4.1 Estimation based on the electromotive force

Equation (3) shows that the relationship between residual capacity of a lead-acid battery and electromotive force [1][3].

When the battery is discharged, the terminal voltage drops because of the internal resistance. Thus, we suppose that the electromotive force is equal to the terminal voltage when the switch turns off in Fig. 3. The electromotive force drops as the resid-

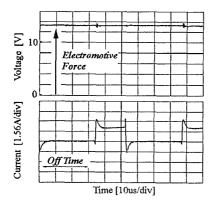


Fig. 3. Waveform of the current and the voltage with switching

ual capacity reduces as shown in Fig.4, in which three kind of discharge experiments are illustrated.

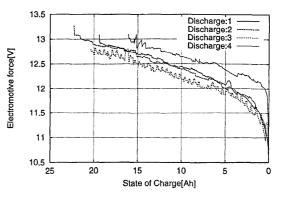


Fig. 4. Relation between electromotive force and the state of charge

The electromotive force suddenly drops, when the residual capacity is almost zero. The reason for this sudden voltage drop is that the shortage of sulfuric acid  $(H_2SO_4)$  and the surface of electrodes are covered with the resulting  $(PbSO_4)$ .

But, before the sudden voltage drop the residual capacity is estimated as (4).

$$C = 100(V_e - 11.7)/(12.8 - 11.7)$$
<sup>(4)</sup>

where  $V_e$  is the electromotive force and C is the residual capacity [3].

From equation (4), we estimated the state of charge with equation (5).

$$C = \alpha V_e + \beta \tag{5}$$

where  $\alpha, \beta$  is the parameter decided by the data of the Discharge:1.

The estimated error is shown in Fig. 5.

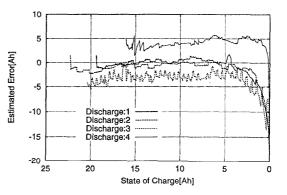


Fig. 5. Estimated error

From Fig. 5, following things are confirmed.

• Around the end of discharge, the estimated error becomes large.

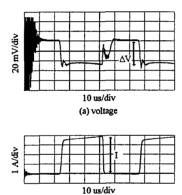
This reason is that the electromotive force suddenly drops at the end of discharge.

• If the discharge current is different, the parameter is different.

This reason is that the electromotive force becomes different at the different discharge current. Thus, in case of the pulse discharging, the estimation of the electromotive force is very difficult, because the non-uniform distribution of the electrolyte around the electrodes causes the electromotive force to vary in a short time.

# 4.2 Estimation based on current integration

If the total integration of the charged current is correctly known, the residual capacity is estimated by integrating the discharge current.[2] The current waveform of the discharged experiment is shown in Fig. 6. It is observed that the current waveform is warped by switching. It is difficult to integrate the discharged current. And even if the amount of the charge is known, the state of the previous discharge has an effect on the residual capacity. This method has difficulty in this point.



(b) current

Fig. 6. waveform of the current and the voltage with switching

# 4.3 Estimation based on the internal resistance

As the amount of discharge increases, the active material in the battery decreases. As a result, the internal resistance increases.

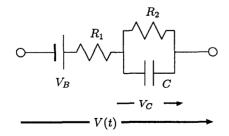


Fig. 7. Equivalent Circuit of Battery

Fig.7 shows an equivalent circuit of a battery that has been proposed in [4] and [5]. In Fig.7  $R_1$  is the electrolyte resistance including the electrode resistance.  $R_2$  is the charge transfer resistance at the interface between the electrode and electrolyte solution, and C is the electric double layer static capacity formed at the interface between the electrode and electrolyte solution.

Using the equivalent circuit shown in Fig.7, the terminal voltage is given as (6).

$$V(t) = V_B - I(R_1 + R_2) + R_2 I \cdot e^{-\overline{CR_1}} - V_C \cdot e^{-\overline{CR_2}}$$
(6)

, where V(t) and I are shown in Fig.6. From (6), the steady state is given as (7).

$$V(t) = V_B - I(R_1 + R_2)$$
(7)

Thus, the internal resistance is obtained as (8).

$$R_1 + R_2 = \Delta V/I \tag{8}$$

In Fig.6, the waveform is in the steady state. Thus, we assume the follow conditions in order to measure the internal resistance.

1. The state of the discharge current is not in the pulse, but we suppose that the maximum value of the discharge current is the pulse current (I).

2. The terminal voltage is not the stable, but we suppose the voltage at the point where the discharge current is maximum, is the value of the stable voltage.

The internal resistance obtained from (8) is shown in Fig.8, from which we see that as the residual capacity decreases, the internal resistance increases. And the internal resistance is in reversibly proportional to the residual capacity. As mentioned before, the residual capacity is able to estimated by the internal resistance to a certain degree.

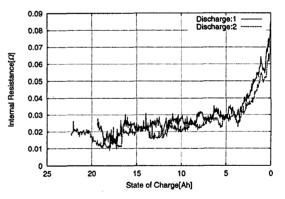


Fig. 8. Relationship between the internal resistance and the state of charge(Discharge:1 and Discharge:2)

The value of the internal resistance can be expressed as a hyperbola of the residual capacity in (9).

$$Ah = \frac{a}{r_{in} - b} + c \tag{9}$$

where a, b, and c is constant. The three parameters are decided by the nonlinear least squares fitting.

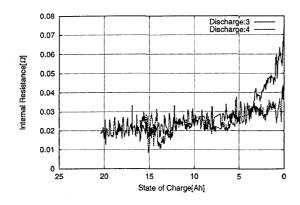
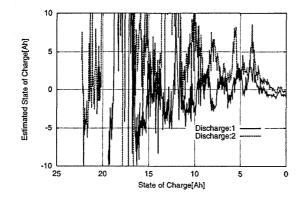


Fig. 9. Relationship between the internal resistance and the state of charge(Discharge:3 and Discharge:4)

# 4.4 Result of estimation

Using (9), we have estimated the state of charge. The estimated error are shown in Fig. 10 and Fig. 11.





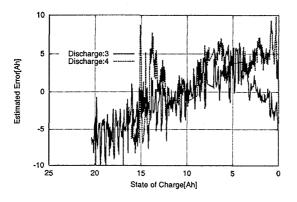


Fig. 11. Estimated error (Discharge:3 and Discharge:4)

But the following are the problems.

• The terminal voltage depends on the temperature. Thus, the adjustment is required when the temperature changes.

• When the residual capacity is about full, the gradient of the internal resistance is not large.

• The value of the internal resistance is imbalance. Thus, the estimation using only the internal resistance is difficult.

# 5 PROPOSED ESTIMATION METHOD FOR THE LEAD-ACID BATTERY

The Fig.4 shows that the electromotive force decreases at some rate before the residual capacity is about 3 Ah. The Fig.8 indicates that the gradient of the internal resistance increases, as the amount of discharge current increases.

We propose the next equation to estimate the residual capacity using the electromotive force and the internal resistance.

$$C_{resi} = \alpha V_e + \beta r_{in} + \gamma \frac{\partial r_{in}}{\partial Ah} + f(T, V_e, r_{in}) + Const(10)$$

 $C_{resi}$ :residual capacity,  $V_e$ :the electromotive force,  $r_{in}$ :internal resistance, T:temperature, Ah:ampere-hc The following is explanation for each term.

• The first term is the electromotive force, and it has relation to the residual capacity until the residual capacity becomes a little.

• The second term is the internal resistance.

• The third term is the gradient of the internal capacity, and it has relation to the residual capacity after the residual capacity becomes a little.

• The fourth is the correctional term for the temperature.

• The fifth term is the constant.

### 5.1 Decision of the parameters

Assuming that the fourth term is negligible, the number of the parameters including the constant term is four.

Thus, the parameters are obtained from the measured data by the least square error (LSE) method.

First, from (10) the residual capacity at nth point can be expressed as (11)

$$C_n = \alpha V_n + \beta R_n + \gamma R'_n + Const \tag{11}$$

Where  $C_n$  is the *n*th residual capacity,  $R_n$  is equal to  $R_1 + R_2$  in (8) and  $R'_n$  is equal to  $\frac{\partial(R_1+R_2)}{\partial Ah}$ .

If n points of measured data are used, (11) is expressed as (12). If n is equal to four, the inverse exists. If n is larger than four, the pseudo inverse matrix is used.

$$\begin{bmatrix} C_1\\ C_2\\ \vdots\\ C_n \end{bmatrix} = \begin{bmatrix} V_1 & R_1 & R'_1 & 1\\ V_2 & R_2 & R'_2 & 1\\ \vdots & \vdots & \vdots & \vdots\\ V_n & R_n & R'_n & 1 \end{bmatrix} \begin{bmatrix} \alpha\\ \beta\\ \gamma\\ Const \end{bmatrix}$$
(12)

5.2 Calculation of the gradient of the internal resistance

It is difficult to find the precise gradient of the internal resistance in Fig.8.

From (9) the gradient of the internal resistance is obtained.

## 5.3 Experiment Result

The results are shown in Fig.12 and Fig.13. The Fig.13 shows the estimation error between the estimated and the actual residual capacity.

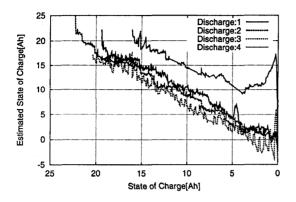


Fig. 12. Estimation of the state of charge from proposed method

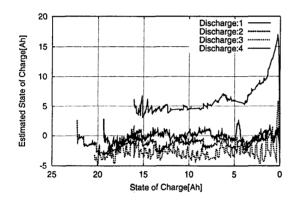


Fig. 13. Estimation error between the proposed method and the state of charge

The Fig.13 shows that the residual capacity is estimated to some extent. And the error of Discharge:1 and Discharge:2 is large as the state of charge becomes small. The internal resistance increases suddenly and the electromotive force decreases suddenly around this region. And the  $\Delta V$ and I were measured by an oscilloscope automatically. Thus, the value of the voltage and the current may contain the switching noise.

But, the error of Discharge:3 and Discharge:4 is large. Especially the error of Discharge:4 is very large. This reason seems the difference of the electromotive force with different discharge current.

#### 6 CONCLUSION

We proposed the residual capacity estimation of the lead-acid battery using the internal resistance and the electromotive force. We confirmed that the residual capacity could be estimated by the proposed equation and the error was below  $\pm 10\%$ . The proposed estimation method seems to have more accuracy  $(\pm 5)$  around the end of discharge.

In future, the residual capacity estimation of the combination battery also becomes important.

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